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TRUTH, OMNISCIENCE, AND CANTORIAN ARGUMENTS: AN EXCHANGE

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INTRODUCTION (GRIM)

In "Logic and Limits of Knowledge and Truth" (*Noûs* 22 (1988), 341–367) I offered a Cantorian argument against a set of all truths, against an approach to possible worlds as maximal sets of propositions, and against omniscience.[1] The basic argument against a set of all truths is as follows:

Suppose there *were* a set **T** of all truths, and consider all subsets of **T** – all members of the power set $\mathscr{P}$**T**. To each element of this power set will correspond a truth. To each set of the power set, for example, a particular truth T_1 either will or will not belong as a member. In either case we will have a truth: that T_1 is a member of that set, or that it is not.

There will then be at least as many truths as there are elements of the power set $\mathscr{P}$**T**. But by Cantor's power set theorem we know that the power set of any set will be larger than the original. There will then be *more* truths than there are members of **T**, and for *any* set of truths **T** there will be some truth left out. There can be no set of all truths.

One thing this gives us, I said, is "a short and sweet Cantorian argument against omniscience." Were there an omniscient being, what that being would know would constitute a set of all truths. But there can be no set of all truths, and so can be no omniscient being.

Such is the setting for the following exchange.[2]

1. PLANTINGA TO GRIM

My main puzzle is this: why do you think the notion of omniscience, or of knowledge having an intrinsic maximum, demands that there be a *set* of all truths? As you point out, it's plausible to think there is no such

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set. Still, there are truths of the sort: **every proposition is true or false** (or if you don't think that's a truth, **every proposition is either true or not-true**). This doesn't require that there be a *set* of all truths: why buy the dogma that quantification essentially involves sets? Perhaps it requires that there be a *property* had by all and only those propositions that are true; but so far as I can see there's no difficulty there. Similarly, then, we may suppose that an omniscient being like God (one that has the maximal degree of knowledge) knows every true proposition and believes no false ones. We must then concede that there is no set of all the propositions God knows. I can't see that there is a problem here for God's knowledge; in the same way, the fact that there is no set of all true propositions constitutes no problem, so far as I can see, for truth.

So I'm inclined to agree that there is no set of all truths, and no recursively enumerable system of all truths. But how does that show that there is a problem for the notion of a being that knows all truths?

2. GRIM TO PLANTINGA

Here are some further thoughts on the issues you raise:

1. The immediate target of the Cantorian argument in the *Noûs* piece is of course a *set* of all truths, or a set of all that an omniscient being would have to know. I think the argument will also apply, however, against any class or collection of all truths as well. In the *Noûs* piece the issue of classes was addressed by pointing out intuitive problems and chronic technical limitations that seem to plague formal class theories. But I also think the issue can be broached more directly — I think something like the Cantorian argument can be constructed against any class, collection, or totality of all truths, and that such an argument can be constructed without any explicit use of the notion of membership . . . .

2. I take your suggestion, however, to be more radical than simply an appeal to some other type of collection 'beyond' sets. What you seem to want to do is to appeal directly to propositional quantification, and of the options available in response to the Cantorian argument I think that is clearly the most plausible.

In the final section of the *Noûs* piece, however, I tried to hedge my claim here a bit:

Is omniscience impossible?
Within any logic we have, I think, the answer is 'yes'.

The immediate problem I see for any appeal to quantification as a way out — *within any logic we have* — is that the only semantics we have for quantification is in terms of *sets*. A set-theoretical semantics for any genuine quantification over all propositions, however, would demand a *set* of all propositions, and any such supposed set will fall victim to precisely the same type of argument levelled against a set of all truths. Within any logic we have there seems to be no place for any genuine quantification over 'all propositions', then, for precisely the same reasons that there is no place for a set of all truths.

One might of course construct a *class*-theoretical semantics for quantification. But if I'm right that the same Cantorian problems face classes, that won't give us an acceptable semantics for quantification over 'all propositions' either.

Given any available semantics for quantification, then — and in that sense 'within any logic we have' — it seems that even appeal to propositional quantification fails to give us an acceptable notion of omniscience. What is a defender of omniscience to do? I see two options here:

(A) One might seriously try to introduce a new and better semantics for quantification. I think this is a genuine possibility, though what I've been able to do in the area so far seems to indicate that a semantics with the requisite features would have to be radically unfamiliar in a number of important ways. (I've talked to Christopher Menzel about this in terms of my notion of 'plenums', but further work remains to be done.) I would also want to emphasize that I think the onus here is on the defender of omniscience or similar notions to actually *produce* such a semantics — an offhand promissory note isn't enough.

(B) One might, on the other hand, propose that we do without formal semantics as we know it. I take such a move to be characteristic of, for example, Boolos' direct appeal to plural noun phrases of our mother tongue in dealing with second-order quantifiers. But with an eye to omniscience I'd say something like this would be a proposal for a notion of omniscience 'without' any logic we have, rather than '*within*'.

I'm also unsure that even an appeal to quantification without standard semantics will work as a response to the Cantorian difficulties at issue regarding 'all truths'. Boolos' proposal seems to me to face some important difficulties, but they may not be relevant here. More relevant, I think, is the prospect that the Cantorian argument against 'all truths' can be constructed using only quantification and some basic intuitions regarding truths — without, in particular, any explicit appeal to sets, classes, or collections of any kind.

3. Consider for example an argument along the following lines, with regard to your suggestion that there might be a *property* had by all and only those propositions that are true:

> Consider any property T which is proposed as applying to all and only truths. Without yet deciding whether T does in fact do what it is supposed to do, we'll call all those things to which T *does* apply t's.
>
> Consider further (1) a property which in fact applies to nothing, and (2) all properties that apply to one or more t's — to one or more of the things to which T in fact applies. [We could technically do without (1) here, but no matter.]
>
> We can now show that there are strictly more properties referred to in (1) and (2) above than there are t's to which our original property T applies. The argument might run as follows:
>
> Suppose any way g of mapping t's one-to-one to properties referred to in (1) and (2) above. Can any such mapping assign a t to *every* such property? No. For consider in particular the property D:
>
> D: the property of being a t to which g(t) — the property it is mapped onto by g — does not apply.
>
> What t could g map onto property D? None. For suppose D is g(t*) for some particular t*; does g(t*) apply to t* or not? If it does, since D applies to only those t for which g(t) does not apply, it does not apply to t*. If it doesn't, since D applies to all those t for which g(t) does not apply, it does apply. Either alternative, then, gives us a contradiction. There is no way of mapping t's one-to-one to properties referred to in (1) and (2) that doesn't leave some property out: there are *more* such properties than there are t's.
>
> Note that for each of the properties referred to in (1) and (2) above, however, there will be a distinct *truth*: a truth of the form 'property p is a property', for example, or 'property p is referred to in (1) or (2)'. There are as many truths as there are such properties, then, but we've also shown that there are more such properties than t's, and thus there must be more truths than there are t's — more truths than our property T, supposed to apply to *all* truths, in fact applies to.

This form of the Cantorian argument, I think, relies in no way on sets or any other explicit notion of collections. It seems to be phrased entirely in terms of quantification and turns simply on notions of truths, of properties, and the fact that the hypothesis of a one-to-one mapping of a certain sort leads to contradiction. It is this type of argument that leads me to believe that Cantorian difficulties regarding 'all truths' go deeper than is sometimes supposed; that the argument applies not only to sets but to all types of collections and that ultimately even quantification fails to offer a way out.

4. Let me turn, however, to another passage in your response:

Still, there are truths of the sort **every proposition is true or false . . .**

What the type of argument offered above seems to suggest, of course, is that there can be no real quantification over 'all propositions'. One casualty of such an argument would be any quantificational outline of omniscience. It must be admitted that another casualty would be 'logical laws' of the form you indicate.

5. By the way, it's sometimes raised as a difficulty that an argument such as the one I've tried to sketch above itself involves what appear to be quantifications over all propositions. I think such an objection could be avoided, however, by judiciously employing scare quotes in order to phrase the entire argument in terms of mere mentions of supposed 'quantifications over all propositions', for example.

3. PLANTINGA TO GRIM

Let me just say this much. Your argument seems to me to show, not that there is a paradox in the idea that there is some property had by all true propositions, but rather that the notion of quantification is not to be understood in terms of sets. Your argument proceeds in terms of mappings, 1-1 mappings, and the like ("Suppose any way *g* of mapping t's one-to-one to properties referred to in (1) and (2) above . . ."); but these notions are ordinarily thought of in terms of sets and functions. Furthermore, you invoke the notion of cardinality; you propose to argue that "there are more properties referred to in (1) and (2) than there are t's to which our original property T applies"; but cardinality too is ordinarily thought of in terms of sets. (And of course we are agreeing from the outset that there is no set of all truths). I don't see any way of stating your argument non set theoretically.

If we think we have to employ the notion of set in order to explain or understand quantification, then some of the problems you mention do indeed arise; but why think that? The semantics ordinarily given for quantification already presupposes the notions of quantification; we speak of the domain D for the quantifier and then say that '(z) Az' is true just in case every member of D has (or is assigned to) A. So the semantics obviously doesn't tell us what quantification is.

Further, it tells us falsehood: what it really tells us is that 'Everything is F' expresses the proposition that each of the things that actually exists is F (and is hence equivalent to a vast conjunction where for each thing in the domain, there is a conjunct to the effect that that thing is F). But that isn't in fact true. If I say 'All dogs are good-natured' the proposition I express could be false even if that conjunction were true. (Consider a state of affairs β in which everything that exists in å (the actual world) exists, plus a few more objects that are evil-tempered dogs; in that state of affairs the proposition I express when I say 'All dogs are good-natured' is false, but the conjunction in question is true.) The proposition to which the semantics directs our attention is materially equivalent to the proposition expressed by 'All dogs are good-natured' but not equivalent to it in the broadly logical sense.

So I don't think we need a set theoretical semantics for quantifiers; I don't think the ones we have actually help us understand quantifiers (they don't get things right with respect to the quantifiers); and if I have to choose between set-theoretical semantics for quantifiers and the notion that it makes perfectly good sense to say, for example, that every proposition is either true or not-true, I'll give up the former.

4. GRIM TO PLANTINGA

You point out that the argument I offered in terms of properties is still phrased using mappings or functions, one-to-one correspondences, and a notion of cardinality, and that these are ordinarily thought of in terms of sets. "I don't see any way," you say, "of stating your argument non set theoretically."

I do. In fact I don't consider the argument to be stated set-theoretically as it stands, strictly speaking; it's a philosophical rather than a formal argument. In order to escape any lingering suggestion of sets, however, we can also outline all of the notions you mention entirely in terms merely of relations — properties applying to pairs of things — and quantification. I don't see any reason for you to object to that; you seem quite happy with both properties and quantification over properties generally.

A relation R gives us a one-to-one mapping from those things that have a property P^1 *into* those things that have a property P^2 just in case:

$$\forall x \forall y[P^1x \;\&\; P^1y \;\&\; \exists z(P^2z \;\&\; Rxz \;\&\; Ryz) \rightarrow x = y] \\ \&\; \forall x[P^1x \rightarrow \exists y \forall z(P^2z \;\&\; Rxz \leftrightarrow z = y)].$$

A relation R gives us a mapping from those things that are P^1 that is one-to-one and *onto* those things that are P^2 just in case (here we merely add a conjunct):

$$\forall x \forall y[P^1x \;\&\; P^1y \;\&\; \exists z(P^2z \;\&\; Rxz \;\&\; Ryz) \rightarrow x = y] \\ \&\; \forall x[P^1x \rightarrow \exists y \forall z(P^2z \;\&\; Rxz \leftrightarrow z = y)] \\ \&\; \forall y[P^2y \rightarrow \exists x P^1x \;\&\; Rxy)].$$ [3]

We can outline cardinality, finally, simply in terms of whether there is or is not a relation that satisfies the first condition but doesn't satisfy the second. I'm not sure that we might not be able to do without even that — I'm not sure we couldn't phrase the argument as a reductio on the assumption of a certain relation, for example, without using any notion of cardinality *within* the argument at all.

I don't agree, then, that the argument depends on importing some kind of major and philosophically foreign set-theoretical machinery. Notions of functions or mappings and one-to-one correspondences *are* central to the argument, but in the sense that these are required they can be outlined purely in terms of relations — or properties applying to pairs of things — and quantification. Cardinality, if we need it at all, can be introduced in a similarly innocuous manner.

You also suggest several other reasons to be unhappy with a set-theoretical semantics for quantification, and end by saying that "if I have to choose between set-theoretical semantics for quantifiers and the notion that it makes perfectly good sense to say, for example, that every proposition is either true or not-true, I'll give up the former."

There may or may not be independent reasons to be unhappy with set-theoretical semantics for quantifiers — I think the points you raise are interesting ones, and I'll want to think about them further. My immediate reaction is that the first point you make does raise a very important question as to what formal semantics can honestly claim or be expected to do, and I'm very sympathetic to the notion that it has sometimes been treated as something that it neither is nor can be. My guess is that the second issue might be handled in a number of ways familiar from different approaches to possible worlds, without any deep threat to set-theoretical semantics. But I could be wrong about that — as I say, I'll want to think about these questions further.

Even if there are independent reasons to be unhappy with set-theoretical semantics, however, I think your final characterization of available options is off the mark. For reasons indicated above, I think sets *aren't* essential to the type of Cantorian argument at issue — the argument can for example be phrased entirely in terms of properties, relations, and quantification. If that's right, however, the basic issue is not one to be settled by some choice between set-theoretical semantics and, say, 'all propositions'. The problems are deeper than that: even abandoning set-theoretical semantics entirely, it seems, wouldn't be *enough* to avoid basic Cantorian difficulties.

5. PLANTINGA TO GRIM

Right: we can define mappings and cardinalities as you suggest, in terms of properties rather than sets. We can then develop the property analogue of Cantor's argument for the conclusion that for any set **S**, P(**S**) (the power set of **S**) > **S** as follows.

Say that **A*** is a **subproperty** of a property **A** iff everything that has **A*** has **A**; and say that the **power property** P(**A**) of a property **A** is the property had by all and only the subproperties of **A**.

Now suppose that for some **A** and its power property P(**A**), there is a mapping (1-1 function) **f** from **A** onto P(**A**). Let **B** be the property of **A** such that a thing **x** has **B** if and only if it does not have **f**(**x**). There must be an inverse image **y** of **B** under **f**; and **y** will have **B** iff **y** does not have **B**, which is too much to put up with.

But if P(**A**) exceeds **A** in cardinality for any **A**, then there won't be a property **A** had by everything; for if there were, it would have a power property that exceeds it in cardinality, which is impossible. So there won't be a property had by every object, and there won't be a property had by every proposition. Hence if we think quantifiers must range over something, either a set or a property, we won't be able to speak of all propositions or of all things.

But of course the Cantorian property argument has premises, and it might be that some of the premises are such that one is less sure of them than of the proposition, e.g., that every proposition is either true or not true, or that everything has the property of self-identity. In particular, one premise of the Cantorian argument as stated is

(α) For any properties **A** and **B** and mapping **f** from **A** onto **B**, there exists the subproperty **C** of **A** such that for any **x**, **x** has **C** if and only if **x** has **A** and **x** does not have **f(x)**

This doesn't seem at all obvious. In particular, suppose there *are* universal properties — **not being a married bachelor**, for example, and suppose the mapping is the identity mapping. Then there exists that subproperty **C** if and only if there is such a property as the property **non-selfexemplification** — which we already know is at best extremely problematic.

So which is more likely: that we can speak of all propositions, properties and the like (and if we can't just how are we understanding (α)?), or that (α) is true? I think I can more easily get along without (α).

One final note. These problems don't seem to me to have anything special to do with omniscience. One who wants to say what omniscience is will have difficulties, of course, in so doing without talking about all propositions. But the same goes for someone who wants to hold that there aren't any married bachelors, or that everything is self-identical. If we accept the Cantorian argument, we shall have to engage in uncomfortable circumlocutions in all these cases, circumlocutions such that it isn't at all clear that we can use them to say what we take to be the truth. But the problem won't be any worse in theology than anywhere else.

The best course though (I think) is to reject (α).

6. GRIM TO PLANTINGA

I think your response to the Cantorian property argument is an interesting one. Here however are some further thoughts on the issue.

Let me start with a reminder as to where we stand.

The Cantorian property argument as you present it is as follows:

Say that A* is a **subproperty** of a property **A** iff everything that has **A*** has **A**; and say that the **power property** P(**A**) of a property **A** is the property had by all and only the subproperties of **A**.

Now suppose that for some **A** and its power property P(**A**), there is a mapping (1-1 function) **f** from **A** onto P(**A**). Let **B** be the property of **A** such that a thing **x** has **B** if and only if it does not have **f**(**x**). There must be an inverse image **y** of **B** under **f**; and **y** will have **B** iff **y** does not have **B**, which is too much to put up with.

As this stands, of course, it is merely an argument that the power property P(**A**) of any property **A** will have a wider extension than does **A**. But if we suppose **A** to be a property had by all properties, or a property had by all things, we will get a contradiction. There can be no such property . . . or so the argument seems to tell us.

The escape you propose here is essentially a denial of the diagonal property required in the argument. Given some favored universal property **A** and a chosen function **f**, what the argument demands is a property **B** 'that is a subproperty of **A** such that a thing **x** has **B** if and only if it does not have **f**(**x**).' But there is no such property. The argument demands that there is, and so is unsound. Or so the strategy goes.

Somewhat more generally, the strategy is to deny any principle such as (α) that tells us that there *will* be a property such as **B**:

(α) For any properties **A** and **B** mapping **f** from **A** onto **B**, there exists the subproperty **C** of **A** such that for any **x**, **x** has **C** if and only if **x** has **A** and **x** does not have **f**(**x**).

(α) "doesn't seem at all obvious," you say. "I think we can easily get along without (α)."

I don't believe that things are by any means that simple. Here I have two fairly informal comments to make, followed by some more formal considerations:

1. As phrased above, I agree, (α) is hardly so obvious as to compel immediate and unwavering assent. The diagonal properties demanded in forms of the argument similar to yours above — properties such as '**B**, a subproperty of **A**, the property 'being a property', that applies to all and only those things which do not have the property **f**(**x**) mapped

onto them by our chosen function **f'** — may similarly lack immediate intuitive appeal.

I think this is largely an artifact of the particular form in which you've presented the Cantorian argument, however. Yours follows standard set-theoretical arguments very closely, complete for example with a notation of 'power property'. The argument becomes formally remote and symbolically prickly as a result, and the diagonal property called for is offered in terms which by their mere technical formality may dull relevant philosophical intuitions.

But the Cantorian argument doesn't *have* to be presented that way. It can, for example, be phrased without any notion of power set or power property at all — on this see "On Sets and Worlds, a Reply to Menzel" . . . .[4] When the argument *is* more smoothly presented, moreover, the diagonal constructed in the argument becomes significantly harder to deny. Consider for example an extract from a form of the argument that appeared earlier in our correspondence:

Consider any property **T** which is proposed as applying to all and only truths. Without yet deciding whether **T** does in fact do what it is supposed to do, we'll call all those things to which **T** *does* apply **t**'s.

Consider further (1) a property which in fact applies to nothing, and (2) all properties that apply to one or more **t**'s — to one or more of the things to which **T** in fact applies . . .

We can now show that there are strictly more properties referred to in (1) and (2) above than there are **t**'s to which our original property **T** applies . . .

Suppose any way **g** of mapping **t**'s one-to-one to properties referred to in (1) and (2) above. Can any such mapping assign a **t** to *every* such property? No. For consider in particular the property **D**:

D: the property of being a **t** to which **g(t)** — the property it is mapped onto by **g** — does not apply.

What **t** could **g** map onto property **D**? None . . .

Consider also the following Cantorian argument:

Can there be a proposition which is genuinely *about* all propositions?

No. For suppose any proposition **P**, and consider all propositions it is about. These we will term **P**-propositions.

Were **P** genuinely about *all* propositions, of course, there would be a one-to-one mapping **f** from **P**-propositions onto propositions simpliciter: a mapping **f** which assigns **P**-propositions to propositions one-to-one and leaves no proposition without an assigned **P**-proposition.

But there can be no such mapping. For suppose there were, and consider all **P**-propositions **p** such that the proposition to which they are assigned by our chosen mapping **f** — their **f(p)** — is *not* about them.

Certainly we can form a proposition about precisely these — using propositional quantification and '**A**' to represent 'about', a proposition of the following form:

$$\forall p((Pp \,\&\, \sim A(f(p))p \rightarrow \ldots p \ldots)$$

Consider any such proposition $\mathbf{P_d}$. What **P**-proposition could **f** map onto it?
None . . .

In the first argument, I think, we have an eminently intuitive property: the property of being a **t** to which a corresponding property we've imagined does not apply. In the second, we have an eminently intuitive proposition. There are, it seems clear, **P**-propositions which won't have a corresponding proposition that happens to be about them. Isn't that itself a proposition that *is* about them?

The general point is this. In order for a strategy of denying the diagonal to prove effective against all offending forms of the Cantorian argument, one would have to deny an entire range of properties and propositions and conditions and truths liable to turn up in a diagonal role. Some of these, I think, will have an intuitive plausibility far stronger than that of the formal construction you offer in your more formal rendition of the argument above.

The diagonals at issue will always involve a function **f** or a relation **R** supposed one-to-one from one batch of things onto another. Such functions or relations alone, we've agreed, seem entirely innocent. But passages such as the following, from other imaginable Cantorian arguments, seem intuitively innocent as well:

f is proposed as a mapping between *known* truths (or truths known by some individual **G**) and all truths. Some known truths will have a corresponding **f**-truth on that mapping that is about them. Some won't. Surely there will be a truth about all those that don't — the truth that they all *are* truths, for example.

f is proposed as a mapping between a group **G** of properties and *all* properties. Some **G**-properties will have corresponding properties by **f** that in fact apply to them. Some won't. Consider all those that don't, and consider the property they thereby share . . .

f is proposed as a mapping between (i) the things a certain fact **F** is a fact about and (ii) all satisfiable conditions. Some things **F** is about will thereby be mapped onto conditions they themsleves satisfy. Some won't. Consider the condition of being something that has an **f**-correlate it doesn't satisfy . . .

Each of these is the diagonal core of a Cantorian argument: against the possibility of all truths being known truths, against any comprehensive grouping of all properties, and against any fact about all satisfiable conditions. When passages such as these are offered step by step and in full philosophical form, I think, the truth, property, and satisfiable condition they call for are *very* intuitive. How, one wants to ask, could there *not* be such a truth, or such a property, or such a condition?

I don't believe, therefore, that the situation is one in which I have a formal argument on my side and you have the intuitions on yours. Although somewhat complex, the Cantorian argument can be presented as a fully philosophical argument with significant intuitive force. I'm also willing to admit that there are at least initial intuitions that somehow truths *should* collect into some totality, or that there *should* be an 'all' to the propositions. What we seem to face, then, is a clash of intuitions. But it *is* a genuine *clash* of intuitions, I think, with genuinely forceful intuitions on both sides.

2. There is also a further difficulty. Consider again the basic structure of our earlier argument against a property had by all and only truths. Essentially:

1. We consider a property **T**, proposed as applying to all and only truths, and call the things it *does* apply to **t**'s.
2. We can show that there are strictly more properties which apply to one or more **t**'s than there are **t**'s. For suppose any way **g** of mapping **t**'s one-to-one onto such properties, and consider in particular the property **D**:

D: the property of being a **t** to which **g**(**t**) does not apply.

What **t** could **g** map onto property **D**? None . . .

3. There are then more properties which apply to one or more **t**'s than there are **t**'s. But for each such property there is a distinct *truth*. Thus there are more truths than **t**'s: contrary to hypothesis, **T** cannot apply to *all* truths.

Here the strategy you propose would have us deny the diagonal property **D**.

If there is no such property, however, the conditions laid down in (2) above are conditions without a corresponding property. 'Being a **t** to which . . .' is merely a stipulation, or a set of conditions, or a specification that fails of propertyhood.

Given any of *these*, however, we will be able to frame a Cantorian argument with much the same form and to precisely the same effect as the original. For at (2) we can show that there are strictly more stipulations, or sets of conditions, or specifications — property-specifying or not — than there are **t**'s. But (in 3) there will be a distinct truth for each of *these*, and thus more truths than **t**'s. Whatever property **T** applies to, we conclude as before, it cannot apply to all truths.

The problem seems to grow. If we deny a supposed diagonal property **D** property-hood, or a proposed diagonal truth **D** truth, or a proposed diagonal proposition **D** propositionality, we'll still want to say what **D** in each case is *instead* — a propertyless condition or a pseudo-truth or a mere logical form short of propositionality or the like. But given *any* answer here, it appears, we'll be able to frame a further Cantorian argument of much the same form and to precisely the same effect as the original.

I think of these as Strengthened forms of the Cantorian argument, analogous in important ways to Strengthened forms of the Liar.

3. Let me also offer some thoughts from a somewhat more formal angle. Here I'll start with a consideration that is admittedly merely suggestive:

The arguments we're dealing with, of course, parallel Cantorian arguments of major importance in set theory and number theory. A strategy of 'denial of the diagonal' will also have a parallel there.

Deny the diagonal in number theory, however, and you face some devastating consequences. The door to Cantor's paradise is immediately closed. Standard construction of the reals from the rationals is blocked, we renounce canonical results regarding the transfinite and fixed point theorems and the like, and the locus classicus of Gödel's and Löb's theorems vanish. Our mathematical world shrinks.

We haven't taken that path in mathematics, and I think there would be general agreement we should not. But why then choose the analogous path here?

4. There are also some significantly stronger arguments from a more formal perspective.

Your proposal, phrased with respect to a particular Cantorian argument, is to deny the existence of a diagonal property stipulated in the course of that argument.

If such a strategy is to apply to offending Cantorian arguments systematically and in general, however, rather than being applied merely *ad hoc* on the whim of the wielder, we need a principle that will tell us *which* diagonal properties, propositions, truths, or conditions to reject. You don't, quite clearly, want to reject *all* diagonals.

Here there is a nested pair of problems.

The first is simply that we've been offered no satisfactory principle of such a sort, and I doubt very much that anyone will in fact be able to produce one.

The second problem is deeper. There are of course deep affinities between the Cantorian results at issue here and certain aspects of the classical paradoxes. If those affinities hold, I think, we can bet that any principle that *was* proposed as an exhaustive condition of what diagonals to accept and what to reject would face a crucial and devastating test case constructed in its own terminology. If so, it's not merely that a comprehensive principle as to which diagonals to accept and which to reject hasn't in fact been offered. If important parallels hold, it's rather that no such principle *could* be offered.

If we are in fact given no principle to guide a strategy of denying the diagonal, I think, such a strategy can only be applied in a manner bound to be rejected as unprincipled and *ad hoc.* If I'm right that no adequate principle *can* be given, of course, any such strategy will moreover be essentially and inescapably *ad hoc.*

5. As you note, a principle which tells us that there *is* the diagonal property your form of the argumet requires is (α):

(α) For any properties **A** and **B** and mapping **f** from **A** onto **B**, there exists the subproperty **C** of **A** such that for any **x**, **x** has **C** if and only if **x** has **A** and **x** does not have **f**(**x**).

You want to deny the diagonal, and so deny (α).

(α) itself, however — like similar principles relevant to other forms of the argument — strictly follows from some very elementary and strongly intuitive assumptions.

In set- and class-theory, the straight analogue for (α) follows essentially from just the power set and separation axioms alone. Lurking just beneath the surface in our correspondence has been an incipient property theory. But which of the following intuitive principles would you deny to *properties*?:

(1) Property Comprehension (or subset, or separation):
If there is a property P^1, there is also the property P^2 that applies to just those P^1 things that are ϕ. (for expressible conditions ϕ)

(2) Power Property:
If there is a property P^1, there is also the property P^2 that applies to the subproperties of P^1. (here we can use your definition of 'subproperty')

Each of these is just as intuitive regarding properties, I think, as regarding sets. There seems no stronger intuitive ground to deny either here than in the case of sets.

The situation is even tighter than this, however.

Of the two principles above, the more plausible candidate for denial is surely (2). But as it turns out, 'power property' *isn't* in fact required for a form of the argument against, say, a notion of truth's totality or a property distinctively characteristic of truths. Comprehension alone is sufficient (on this once again I call your attention to "On Sets and Worlds").

In order to escape a Cantorian argument and save a property of all and only truths, say, we would have to put major limitations on any principle of comprehension for properties.

That is of course essentially what was done in avoiding Russell's and Cantor's paradoxes by the creation of ZF set theory. Here our restrictions on Comprehension would have to be still tighter, however, tied to properties, propositions, truths, and the like.

The result of such a restriction in ZF, however, is an explicit sacrifice of Cantor's 'set of all sets'. Try a similar restriction in property theory, I think, and comprehension here will fail to countenance any property of all properties, any proposition regarding all propositions, any truth about all truths, and the like.

The point can also be put another way. If one *could* specify a restriction on property comprehension which would intuitively escape our Cantorian arguments throughout and yet would allow for a property of all properties, a proposition about all propositions, and the like, we could predictably read off it a form of set theory that would escape

Cantor's paradox while including Cantor's set. I'm sure that the set theorists would love to hear about it.

6. Finally, however, let me agree on an important point. The problems at issue here are *not* unique to theology, and I've never said they were. These are problems quite generally regarding metaphysical and epistemological notions of the widest scope. Such concepts appear in philosophical theology as well as elsewhere.

At the end of your comments, you suggest that Cantorian arguments may force us to "uncomfortable circumlocutions" in a wide range of cases. I want to emphasize that the central problems at issue here seem to me far deeper than that. These are *real* conceptual problems, as solid as contradiction. They're problems for metaphysics and epistemology generally, rather than for philosophical theology alone, but they're not problems that any circumlocution, however uncomfortable, is genuinely going to resolve.

7. PLANTINGA TO GRIM

I think we may have gone about as far as we can go here; from here on we may find ourselves simply repeating ourselves; perhaps we shall just have to agree to disagree. I'd like to summarize briefly how I see the situation as a result of our discussion, and introduce one additional consideration.

First, a Cantorian argument for the conclusion that no propositions are about all propositions or properties (that no propositions are genuinely universal) will typically involve a diagonal property or proposition; I propose that in every case it will be less unlovely, intuitively speaking, to deny the relevant diagonal premise than to accede in the conclusion. (More on the unloveliness of the conclusion below.) You remark, quite correctly, that not all diagonal propositions and properties are to be rejected, and it seems at best extremely difficult and maybe impossible to give general directions as to which diagonal propositions and properties to accept and which to reject. You go on to say, however, that the rejection of a given diagonal premise will be "unprincipled", "at the whim of the wielder" and "ad hoc". This doesn't seem to me to follow. First, the course I suggest is not unprincipled.

The principle involved is this: of a number of propositions that together lead to a contradiction by impeccable argument forms, give up the propositions that have the least intuitive support. In the examples I can think of, it seems to me much more intuitive to reject the relatively complex and obscure diagonal premise than to reject the proposition that some propositions are genuinely universal. Obviously this also isn't a matter of whim; and while it is ad hoc, it is not ad hoc in an objectionable sense. It would indeed be nice to have such general directions; but here as in most areas of philosophy and logic we don't have anything like a satisfactory algorithm for determining what will and what won't get us into trouble. That's just part of the human condition.

I do agree with you, though, that there is indeed a *cost* here. It seems as if there *should* be the diagonal properties or propositions involved. So I agree with you when you say "But it is a genuine *clash* of intuitions, I think, with genuinely forceful intuitions on both sides." What we have here, after all, is a *paradox*, and any way out of a genuine paradox exacts a price. But the intuitive support for the existence of genuinely universal propositions (as I see it) is stronger than for the relevant diagonal premises; so the price is right.

Finally, (and here I'm introducing something new, not just summarizing) it seems to me that there is self-referential trouble with your position; it is in a certain way self-defeating. First, it seems hard to see how to *state* your argument. Consider, for example, the main statement of your Cantorian argument on p. 52 and also on p. 59. This argument begins:

Consider any property T which is proposed as applying to all and only truths.

Then it alleges that any such property T will have some further property Q. But then the next step of the argument must be something like:

So any property T which is proposed as applying to all and only truths will have Q.

And that is a quantification over all properties — which, according to your conclusion, is illicit. So how is the argument to be stated?

Perhaps as follows. *I* believe that there are propositions that are genuinely universal; i.e., that quantify over all propositions or properties. *You* propose various premises which I am inclined to some degree to accept and which together (and by way of argument forms I accept) yield a conclusion. You yourself don't take any responsibility for any of the premises, of course; your argument is strictly dialectical. (It isn't even a reductio, because your conclusion implies not that the supposition to be reduced to absurdity is absurd, but that it doesn't so much as exist.) But you are enabling *me* to apprehend an argument which shows that something I believe is mistaken.

But what is the conclusion to be drawn? That is, what conclusion is it that I am supposed to draw: what is the conclusion you suggest is the right one for me to draw, from the argument you suggest? (This conclusion is also one you will have presumably drawn upon offering the same argument to yourself.) Now here we must be careful: it is tempting, of course, to say that the conclusion is that

There are no genuinely universal propositions.

But of course that is itself a genuinely universal proposition, stating as it does that every proposition is non(genuinely universal).

You suggest that we can avoid the problem here by judicious use of scare quotes. ("It's sometimes raised as a difficulty that an argument such as the one I've tried to sketch above itself involves what appear to be quantifications over all propositions. I think such an objection could be avoided, however, by judiciously employing scare quotes in order to phrase the entire argument in terms of mere mentions of supposed 'quantifications . . .'.") But how is that supposed to work? The conclusion will be expressed in a sentence, presumably one involving scare quotes. Either that sentence expresses a proposition or it does not. If it does not, we won't make any advance by using the sentence; if it does, we should be able to remove the scare quotes. But how can we remove the scare quotes? What is the conclusion of the argument, straightforwardly stated?

The use of quotes suggests that the conclusion has something to do with some phrase, or sentence, or perhaps some linguistic term. But what would that be? Consider, for example, the sentence

(a) Every proposition is either true or not-true

which is in some way unsatisfactory on this version of your view. What is the problem? It isn't that this form of words expresses a proposition, which proposition is necessarily false: for your conclusion, put my way, is that there aren't any genuinely universal propositions — there isn't any such thing as quantification over all propositions or properties. Shall we say that (a) is ill-formed, does not conform to the rules of English sentence formation? That seems clearly false. Shall we say that (a) does not (contrary to appearances) succeed in expressing a proposition? That can't be right; for *that* conclusion is again a genuinely universal proposition, saying of each proposition that it has the property of not being expressed by (a). Shall we say that (a) is meaningless? That isn't right either; we certainly understand it, and can deduce from it (from the proposition it expresses) with the other premises you suggest the conclusions needed to make the Cantorian argument work. So what would be the problem with (a)?

And in any event, the conclusion of your argument, I take it, isn't really about linguistic items, expressions of English or of any other language. It is really an ontological conclusion about propositions, saying that there is a certain kind of proposition — the kind *genuinely universal* — such that once we get really clear about that kind, we see that it can't have any examples. That's really the conclusion; but that conclusion, sadly enough, is also self-referentially incoherent in that it is an example of the kind it says has no examples; it quantifies over propositions generally, saying that each of them lacks the property of being genuinely universal.

So the right course, as I see it, is to persevere in the original and intuitive view that there are indeed propositions that quantify over all propositions: for example, *every proposition is either true or not true*, and *for any propositions P and Q, if, if P then Q, and P, then Q*. The alternative seems to be to say that there simply are no such propositions (no propositions that quantify over all propositions or properties): but that proposition seems to be self referentially absurd, and also (given a couple of other plausible premises) necessarily false. We should then try to avoid paradox by refusing to assert the premises that (together with the above) yield paradox; we don't get into trouble, after all, simply by making the above assertion. And one reason for resisting some of the premises of those paradox-concluding arguments is just that, together with other things that seem acceptable, they lead to the conclusion that no propositions are about all propositions, which is itself deeply paradoxical.

It must be granted that some of those premises seem initially innocent, and even to have a certain degree of intuitive warrant. The conclusion has to be, I think, that they don't have as much intuitive warrant as does the proposition that some propositions are indeed about all propositions.

Finally, I return to the point that the problem here, insofar as it is a problem, isn't really a problem for traditional theology. It is a general problem with a life of its own; and you don't get a problem for theology by taking a problem with a life of its own and nailing it to theology. If there aren't any really general propositions (and notice that the antecedent looks like a really general proposition) then the thesis that God is omniscient will have to be stated in some other way, as will such paradigms of good sense as that no proposition is both true and false.

8. GRIM TO PLANTINGA

At the core of the issue, we agree, is a clash of intuitions. On the one side are the intuitions that fuel the Cantorian argument in its various forms. On the other side is the lingering feeling that there nonetheless somehow *ought* to be some totality of all truths or of all propositions.

What you propose as a way out is that we pick and choose, argument by argument, which to give up: the totality of truths or propositions or things known that the argument explicitly attacks, or the diagonal truth or proposition or thing known that it uses to attack that totality. Our universal principle, you propose, is this: case by case we let intuition be our guide. I'm afraid that seems to me to be neither universal nor a genuine principle.

I also don't want the basic clash of intuitions to be misportrayed. If we treat this as a choice, the choice is not between (1) the intuitive appeal of an idea of omniscience, say, and (2) the intuitive appeal of an awkwardly phrased diagonal 'piece of knowledge' proposed in the Cantorian argument against omniscience. When properly understood, the choice is rather between (1) the intuitive appeal of an idea of omniscience and (2) some very basic principles regarding totalities, truth, and knowledge. Given those basic intuitive principles, it *follows* that there will be a diagonal 'piece of knowledge' of the sort the argument calls for. One can't then *just* deny the diagonal; one would have to deny one or more of the intuitive principles behind it as well.

Consider for example the argument that there can be no totality of the things that an omniscient being would have to know. Here the Cantorian argument has us envisage sub-totalities of that supposed totality, and proceeds by showing that for any proposed one-to-one mapping **f** from individual things known to subtotalities of the whole there will be some subtotality left out. In particular, perhaps, we envisage that 'diagonal' bunch of individual things known which do not appear in the subtotalities to which **f** maps them.

Are we to deny that there really *is* such a diagonal subtotality? I simply don't see how. We started by supposing a certain totality — a big bunch of things. Once we have those, the 'subtotality' at issue is simply a bunch of things we already had. We didn't create them. The most we've done is to specify them, without ambiguity, one way among others, in terms of the mapping **f**.

Denial of the diagonal at this stage doesn't thus seem very promising. Since there *are* these things, however, it seems there must be a *truth* about these things. Otherwise truth would be something far cheaper and more paltry than we take it to be. Truth wouldn't be the whole story: there would be things out there without any truths about them.[5]

If there is a *truth* about these things, however, an omniscient being would have to know that truth. Otherwise omniscience would be something far cheaper and more paltry than we take it to be: an omniscient being would be said to know everything, perhaps, even if there are some truths he doesn't know.

Denying the diagonal in a case like this is thus not simply a matter of denying some one awkwardly phrased piece of proposed knowledge. To deny the diagonal we must deny that if we have the things of a group we still have them when we talk about sub-groups, perhaps, or to deny that anything there is is something there is a truth about, or that knowledge is knowledge of truths, or that universal knowledge would be knowledge of all truths.

Another basic difficulty in selectively denying diagonals, which it seems to me you haven't addressed, is the strengthened Cantorian argument: the problem of the 'reappearing diagonal'. If in some case we choose to deny that there *is* a diagonal truth or proposition of some sort — a proposition about all those propositions which are not about the propositions mapped onto them by a particular function **f**, for example — we will have to do so by claiming that the specification at issue fails to give us a proposition, or that the diagonal condition fails of propositionhood, or the like. But then there will be a Cantorian argument parallel to the original which relies merely on the fact that for every *specification* or *condition* there will be a truth or proposition. We will thus *still* have an argument which shows that there can be no totality of truths or propositions or the like. Denying the diagonal in the arguments at issue simply doesn't seem to work.

Let me turn briefly to your new point, though I think that a complete treatment of this issue would take us well beyond our exchange — and perhaps our abilities — here.

The purest form of the argument, I think, is one which you represent with beautiful clarity:

> *You* propose various premises which I am inclined to some degree to accept and which together (and by way of argument forms I accept) yield a contradiction. You yourself don't take any responsibility for any of the premises, of course; your argument is strictly dialectical . . . . But you are enabling *me* to apprehend an argument which shows that something I believe is mistaken. (pp. 66–67)

But given such an argument, you ask, what positive conclusion should be drawn? These, as you rightly point out, are dangerous waters. I can't claim to have navigated them all, nor can I claim to be able to anticipate all possible dangers. Let me nonetheless sketch some ideas:

One possibility, of course, is that I *shouldn't* attempt a positive conclusion. Perhaps it is enough for me to guide the opposition into their own conceptual mazes of consternation and confusion. That would teach them a lesson, even if not a propositional one.

I'm not yet convinced that we can't have some kind of positive conclusion, however. In the past I've proposed phrasing such a conclusion, in at least some cases, by using scare quotes or some other means of indirect speech. At this point you say that if our conclusion expresses a proposition "we should be able to remove the scare quotes." But I'm not sure why you think that. Scare quotes serve a variety of important and little-understood functions, and it may be that sometimes what we want to say can only be expressed by such means. Why think that they are always avoidable? At this point you also say that the use of quotes "suggests that the conclusion has something to do with some phrase, or sentence, or perhaps some linguistic term." But isn't that buying into a fairly shallow logician's notion of quotation as naming? (As I remember, Anscombe and Haack have some fairly telling points to make against such a treatment.)[6]

For the moment, however, let me try to avoid the difficulties of quotes by proposing another possibility for a positive conclusion.

A simple example helps. We can convince ourselves, I think, that the concept of round squares is an incoherent one. It is tempting to conclude on that basis that round squares don't exist. But this last position brings with it the well-known philosophical difficulties of negative existentials. Perhaps the apparent difficulties there are *merely* apparent. But at any rate we can avoid them by stopping with our first claim: that the concept of round squares is an incoherent one.

Perhaps that is how we should phrase our positive conclusion here as well: the concept of omniscience is an incoherent concept, as is the notion of a totality of truth or of a proposition about all propositions. Having convinced ourselves that the notion of a proposition about all propositions is an incoherent one, we are tempted to conclude that no propositions are genuinely universal. The phrasing of this last position brings with it all the philosophical difficulties you point out. But perhaps we could avoid them, while still having a positive conclusion, by stopping with our first claim: that the concepts at issue are incoherent ones.

My fallback and first love remains the pure form of the argument above, offered without positive conclusion. If a positive conclusion is demanded, this suggestion is perhaps worth a try. It must be added immediately, however, that we'll be able to take this suggestion seriously only if we're willing to give up a few things: at least (1) a Russellian treatment of definite descriptions and (2) the idea that simple predications somehow involve hidden quantifications. But it is perhaps time we gave up those anyway.

9. PLANTINGA TO GRIM

I think we are making progress, but perhaps we are also approaching the ends of our respective ropes; here is my final salvo.

First, I reiterate that the problem we have on our hands, whatever exactly it is, isn't really a problem about omniscience. Omniscience (above, p. 50) should be thought of as a *maximal degree of knowledge*, or better, as *maximal perfection with respect to knowledge.* Historically, this perfection has often been understood in such a way that a being **x** is omniscient only if for every proposition **p**, **x** knows whether **p** is true. (I understand it that way myself.) This of course involves quantification over all propositions. Now you suggest that there is a problem here: we *can't* quantify over all propositions, because Cantorian arguments show that there aren't any propositionally universal propositions (propositions about all propositions – 'universal propositions' for short), and also aren't any properties had by all and only propositions. (Note, by the way that each of these conclusions is itself a universal proposition.) But suppose you are right: what we have, then, is a difficulty, not for *omniscience* as such, but for one way of explicating omniscience, one way of saying what this maximal perfection with respect to knowledge is. A person who agrees with you will then be obliged to explain this maximal perfection in some other way; but she won't be obliged, at any rate just by these considerations, to give up the notion of omiscience itself.

Second, you and I agree that what we have here is a clash of intuitions; but I am not quite satisfied with your outlining of the attractions on each side. You put it like this:

> . . . the choice is not between (1) the intuitive appeal of an idea of omniscience, say, and (2) the intuitive appeal of an awkwardly phrased diagonal 'piece of knowledge' proposed in the Cantorian argument against omniscience. When properly understood, the choice is rather between (1) the intuitive appeal to an idea of omniscience, and (2) some very basic principles regarding totalities, truth and knowledge. Given those basic intuitive principles, it *follows* that there will be a diagonal 'piece of knowledge' of the sort the argument calls for (pp. 69–70).

You also suggest on that same page that on my side of the scales there is in addition "the lingering feeling that there somehow *ought* to be some totality of all truths or of all propositions".

But the attraction of my view here is not that it enables us to save omniscience; omniscience isn't in any danger in any event (what is in danger, as I just argued, would be at most a certain way of explicating omniscience). Nor is the main attraction a lingering feeling that there must somehow be a totality of propositions. Perhaps there is no such totality (a set, a class) of propositions; sets and classes are a real problem anyway. What has the most powerful intuitive force behind it, as I see it, is rather the idea that there are universal propositions (and properties): such propositions, for example, as

(1) Every proposition is either true or not true,

and

(2) There are no genuinely universal propositions.

As I see it, (1) is an obvious truth; there obviously is such a proposition as (1) and it is obviously true. There also seems obviously to be such a proposition as (2) (even if, as I think, it is false), and (2) seems initially to represent your position. "Initially", I say, because (2) seems to be self-referentially incoherent; it is or implies by ordinary logic, the universal proposition

(3) for every proposition **p** there is a proposition **q** such that **p** is not about **q**.

Third, (and most important): as you point out, if we propose to reject a premise in a Cantorian argument, we are of course committed to rejecting any propositions that entail that premise; among the propositions entailing such premises, you say, are "some very basic principles regarding totalities, truth and knowledge"; and you add that it will be hard to reject these. But here is my problem. What will these principles be? In particular, won't they themselves have to be (or include) universal propositions, and hence be such that on your view there really aren't any such things? This is the question I'd like to explore a bit further.

Consider, for example, the argument you offer on p. 59 for the conclusion that there are no universal propositions. Suppose, you say, there were such a proposition **P** (a proposition about all propositions) and consider **P**-propositions: the propositions **P** is about. Then you say

Were **P** genuinely about *all* propositions, of course, there would be a one-to-one mapping **f** from **P**-propositions onto propositions simpliciter . . .

And then you argue that there can't be any such mapping. For suppose there were; then there would have to be a proposition **q** about exactly those propositions **p** which are such that **f(p)** is not about **p**. But then consider the inverse image of **q** under the mapping **f** (call it '**r**'). Is **q** about **r**? Well, it is if and only if it isn't — not a pretty picture.

Here we have two premises:

(4) For any proposition **p**, if **p** is about all propositions, then there is a 1-1 mapping from the propositions **p** is about onto propositions generally.

and

(5) For any function **f**, if **f** is 1-1 and from propositions onto propositions, then there is a proposition **q** about exactly those propositions **p** such that **f(p)** is not about **p**.

Now I want to make 3 points about this argument. First, it initially looks as if you are *endorsing* (4) and (5), or at any rate *recommending* them to me and others. You propose it will be hard to reject them, that they have considerable intuitive force and considerable intuitive claim upon us. But of course on your own view (putting it my way) there really *aren't* any such propositions as (4) and (5), since each involves quantification over all propositions. ((4) is a universal proposition, and both its antecedent and consequent involve universal propositions, as do the antecedent and consequent of (5).) So what do you propose to do with (4) and (5)? What stance do you take with respect to them? Can you conscientiously recommend them to me if you really think there aren't any such propositions, but only, so to speak, a confusion in the dialectical space I take them to occupy? Well, perhaps, in accord with your favorite way of understanding Cantorian arguments (as outlined on p. 71) you aren't *yourself* accepting (4) and (5), but simply proposing to me that if *I* believe that there are any universal propositions at all, then I should also believe (4) and (5); this will land me in hot water; so I shouldn't believe that there are any universal propositions. I doubt that you can properly recommend this to me, because your recommendation presupposes that there are universal propositions: what you say implies that it is possible to believe (4) and (5); but on your own view, it isn't possible, there being no such things to believe. So perhaps you will have to find some other way of stating what you propose.

Second (and waiving the first difficulty) I want to stray from the main topic here (the question how you stand related to (4) and (5)) and ask parenthetically why we should think that if I believe there are universal propositions — e.g., such a law of logic as

(6) For any proposition **p**, **p** is not both true and false

I should also believe (4) and (5)? Why must I believe that if there is such a proposition as (6) (one which is about all propositions) then (4) and (5) are true? I don't dispute (4): I don't dispute that there is an identity mapping on propositions, and if there is, then (4) is true. But what about (5)? Is there really a proposition which is about exactly those propositions **p** such that **f(p)** is not about **p**? Take **f** to be the identity map: is there really a proposition about just those propositions that are not about themselves? Well, it certainly doesn't *look* as if there is such a proposition. If there were, it would presumably have the form

(7) For any proposition **p**, if **p** is about exactly those propositions not about themselves, then . . . .

Aboutness is a frail reed and our grasp of it a bit tenuous, but a proposition like (7) seems to be about *all* propositions, predicating of each property of being such that if it is not about itself, then it is . . . . So such a proposition isn't about *only* those propositions that are not about themselves, unless no proposition is about itself. And the same holds for (5), the premise of your argument. Such a proposition would presumably have the form you give it on p. 60:

(8) For any proposition **p**, if **p** is a **P** proposition and **f(p)** is not about **p**, then . . . **p** . . . .

But a proposition of this form is not, as (5) requires, about only those propositions **p** such that **f(p)** is not about **p** (unless all propositions meet that condition): it seems instead to be about every proposition, predicating of each the property of being such that if it is a **p** proposition and **f(p)** is not about it, then . . . **p** . . . .

So I am not at all inclined to accept (5). (6) is obvious, has a sort of utter seethroughability, a luminous indisputability, an evident lustre, as Locke says. The suggestion that it is not only not true but in fact not even existent is a sort of affront to the intellect – a vastly greater affront than the rejection of (5), which has at best a marginal plausibility. Accordingly, I don't think this argument against the existence of universal propositions is at all powerful.

But now back to the main topic: your relation to (4) and (5); there is a really fascinating point here. Let's briefly recapitulate. I believe that both (1) and (6) are true and also that there is an omniscient being (God), and I take this latter to imply that God knows, for every proposition **p**, whether **p** is true. You propose to make trouble for these beliefs by way of citing Cantorian arguments. As we have seen, there is considerable (self-referential) difficulty in construing the relationship in which you stand to these arguments. You suggest several possibilities. One possibility is that you yourself accept the conclusion, but can't state it without using quotes. But then I really don't understand the conclusion (because I don't know how the quotes are supposed to be working here) and therefore cannot accept it. A second suggestion you make is that you accept the conclusion, and the conclusion is that the concept of a universal proposition (one about every proposition) is incoherent, just as the concept of a round square is incoherent. But what is it for a concept to be incoherent? I see what it is for a concept to be necessarily unexemplified, as is the case with the concept of a round square. But if we say that the conclusion of the Cantorian argument is that this concept of a universal proposition is necessarily not exemplified, then you are again in self-referential trouble. For this concept is necessarily unexemplified only if it is necessarily true that there aren't any universal propositions – i.e., only if necessarily, for every proposition **p** there is a proposition **q** such that **p** is not about **q**. And of course that is itself (the necessitation of) a universal proposition.

Your favorite way of construing the argument (p. 71), however, is still different. Here the idea is that you don't accept or take responsibility for the premises or conclusion of these Cantorian arguments, but instead are only trying to enable me (and others) to apprehend an argument that shows that we have fallen into incoherence in taking ourselves to believe (1) and (6). Now how, exactly, are we to construe this? You don't take responsibility for any premises or the conclusion: you are simply trying to bring about a certain effect in me. But what effect? Well, you apparently hope to get me to believe that there are propositions I am inclined to accept from which it follows that there aren't any universal propositions. (Here suppose we waive the problem that I am not, in fact, at all strongly inclined to believe (5) and the propositions like it to which you direct me.)

But it looks as if these premises, if they are to show that there aren't any universal propositions, will have to contain a universal proposition among them. The conclusion will be or be equivalent to

(9) Every proposition is non-universal, i.e., for every proposition **p**, there is a proposition **q** such that **p** is not about **q**.

and to deduce this conclusion, it looks as if we shall need at least one universal premise — premises like your (4) and (5) for example. So I am now supposed to see that a universal premise I accept entails that there are no universal propositions. I suppose we agree further that if a proposition is a universal proposition, then it is *essentially* a universal proposition, couldn't have failed to be a universal proposition. But then that premise — the universal premise that entails that there aren't any universal propositions — also seems to entail that it doesn't itself exist!

So if you are right, I am in a dialectical situation peculiar *in excelsis.* I believe something **x** from which it follows that **x** isn't merely not true, but doesn't even exist! But then shouldn't I stop believing **x**? Don't I have a proof that **x** is not true? If **x** entails that it doesn't exist, then **x** can't possibly be true. And the same would hold for any set of premises sufficient for a Cantorian argument against universal propositions: they can't all be true because taken together they imply that one of them does not exist.

By way of conclusion: the upshot, so I think, is that I have no reason at all to stop believing (1), or (6), or that there is a being that knows, for every proposition, whether it is true. For any premises that imply that there is no such proposition or being also imply that they themselves do not exist. If they are all true, therefore, they do not all exist; if they do not all exist, they are not all true; therefore they are not all true.

A bit more fully: suppose there is a Cantorian argument **C** whose premises entail that there are no universal propositions. Now **C** may have superfluous premises; so note that for any such argument **C** there is a minimal argument **C*** whose premises are a subset **S** of **C**'s and are such that no proper subset of **S** entails that there are no universal propositions; **C*** will be valid if and only if **C** is. **C*** will contain at least one universal premise; so if the premises of **C*** are all true, they don't all exist. But by hypothesis they all exist; hence they aren't all true; hence there is no sound Cantorian argument against universal propositions!

10. GRIM TO PLANTINGA

There are important points here. Let me try to address some of the main ones:

1. It's true that the Cantorian problems at issue are not first and foremost problems for omniscience per se. They seem to arise as quite general epistemological and metaphysical problems whenever we try to bring together unrestricted notions of truth, knowledge, and totality. But that doesn't mean they *aren't* problems for omniscience.

Omniscience is standardly glossed as being 'all-knowing' or 'knowing everything', precisely as its 'omni' would suggest. If there is no 'everything' of the relevant type to know, there can be no omniscience as standardly glossed.

You suggest that we understand omniscience as 'a maximal degree of knowledge' or as 'maximal perfection with respect to knowledge' (above, p. 73). (Isn't 'maximal perfection' a bit redundant?) But should it turn out that for any degree of knowledge there must be a greater, it would appear that there *can* be no 'maximal perfection' with respect to knowledge — and thus no omniscience as you suggest we understand it.

None of that means that a theist cannot continue to think of God's knowledge as suitably divine in kind or extent. But it may mean that such knowledge — however divine — cannot literally qualify as omniscience in either of the senses outlined.

2. The most fascinating part of your last response is the final argument, to the effect that there can be no sound Cantorian argument

against universal propositions. I think that is a beautiful and deep piece of work.

Were there a sound Cantorian argument with the conclusion that there can be no universal propositions — so the argument goes — it would require at least one universal proposition as a premise. But if sound, its conclusion would be true, and thus there could be no such proposition. If sound its premises would *not* all be true, and thus it would *not* be sound. There can then be no sound Cantorian argument with the conclusion that there can be no universal propositions.

Very nice.

In the end, however, I think this argument simply reinforces some of the points we've already agreed on above. We've already recognized that there are (self-defeating) difficulties with the idea of a straightforward positive proposition to the effect that there can be no universal propositions. It should therefore not be surprising that there would be (self-defeating) problems for the claim that there was some argument, Cantorian or of any other kind, which demonstrated such a proposition.

Here as before I think I have to turn to less direct and more deviously dialectical characterizations of what it is the arguments at issue really do. Contrary to the characterization you give, I'm *not* trying to get you to envisage and accept an argument with some universal premise and a universal conclusion to the effect that there are no universal propositions. You characterize yourself as holding certain beliefs. I merely help you to see that you are thereby led to confusion and consternation.

3. I don't think, then, that your final argument shows what you think it does.

Suppose we grant that any Cantorian argument with a propositional conclusion to the effect that there are no universal propositions would have to have some universal proposition as a premise. By the argument above, there can then be no sound Cantorian argument to that effect.

But interestingly enough, it doesn't seem to follow that universal propositions are then safe from Cantorian arguments — that you "have no reason at all to stop believing (1), or (6) . . ." (p. 78) or other universal propositions of your choice.

To see this, consider again the standard structure of the Cantorian arguments throughout. Someone proposes some set **T** as a set of all truths, or claims that some being **B** is omniscient, or offers a candidate **p** as a genuinely universal proposition. But then consider the power set of **T**, or of what **B** knows, or of the propositions **p** is about. (Although convenient, we've seen that the use of sets here is strictly inessential.) For each of the elements of that particular power set there will be a truth, or something **B** ought to know, or something **p** ought to be about. But then there will be too many of these to put in 1-to-1 correspondence with the truths **T** does contain, or the things **B** does know, or the propositions **p** is about: there are truths or bits of knowledge or propositions that the proposed candidate leaves out.

Now the interesting thing about that core argument is that it is written *in the particular* – it deals simply with a single candidate set **T** or being **B** or proposition **p**. I think, moreover, that it can in each case be written *purely* in the particular, without any universal propositions at all. If that is true, such a form of argument will continue to cause problems for some of the things you claim to believe even if it's also the case that there is no classicial deduction of a universal proposition denying the existence of universal propositions. At some point you will find yourself considering some purportedly universal proposition or totality of truths or omniscient being and we will put that particular candidate through an argument of this form and it will come up short.

Because we can 'see' that this kind of trouble is bound to come up, it is of course tempting to say that such an argument will hold for any arbitrary candidate, and thus must hold for them all. It is tempting, in other words, to read **T**, **B**, and **p** as variables and then finish with a universal generalization. 'There can be no universal propositions.' But that's the point at which we (or at least I) would get into trouble, announcing a positive position which would *also* be subject to the type of argument at issue, and thus a position which – as you point out – could not be the conclusion of a sound argument of this type.

What we should conclude, I think, is that Cantorian arguments are indeed very peculiar, tempting us in some cases to try to draw universal conclusions that they themselves show us cannot be drawn. That is something that your argument points up magnificently. But the fact that they cannot be characterized as universal derivations of universal conclusions in such cases doesn't mean they are somehow harmless – that you "have no reason at all to stop believing (1), or (6), or that there is a being that knows, for every proposition, whether it is true." It's clear that particular candidates for universal propositions or for totalities of truth or knowledge will still be vulnerable to particularized arguments of the form above.

4. If this is right, I think, someone of your convictions will be forced back to the simpler strategy of denying the diagonal in the particular arguments with which someone of my propensities assails you. This is the strategy you take with regard to (5), for example. In general, I think it is the strategy you *should* take, though it will be easier in some cases than in others.

Your approach with respect to (5) is to deny the existence of a proposition about precisely those propositions **p** such that **f(p)** is not about them. The reason you give is that any such proposition would presumably be of the form

$$\forall p((Pp \,\&\, {\sim}A(f(p))\,p \rightarrow \ldots p \ldots),$$

read as

(8) For any proposition **p**, if **p** is a **P** proposition and **f(p)** is not about **p**, then . . . **p** . . . .

But this, you insist, *isn't* about just those propositions. It's about *all* propositions.

'Aboutness' may, as you say, be a frail reed (p. 76). But I don't think it's *that* easily bent. The type of approach you outline here, as you probably know, leads directly to Russell's claim that all propositions are about the universe as a whole. Our standard intuitions would suggest otherwise — we'd normally think that the last sentence is about an approach that leads to Russell, for example, but is not about Michael Jackson's nose job. I agree that our grasp of 'about' may be a bit tenuous, but it certainly doesn't seem to me to be *that* tenuous.

It also seems to me that the strategy you pursue with regard to (5) can be easily circumvented. At the point in the argument at which you raise the issue above, you don't seem to have any objection to the idea that there *are* those propositions **p** such that **f(p)** is not about **p**. At that stage in the argument we already know, moreover, that any 1-to-1 correspondence between propositions and groups will leave this bunch out. What you deny, as outlined above, is my way of getting from that bunch of propositions to a particular proposition; what you deny is that there will be any proposition about those and only those propositions.

But there are lots of other ways of getting to a particular proposition from here. Consider again that bunch of propositions **p** such that **f(p)** is not about **p**. Is there not a property which precisely these propositions share, and a proposition to the effect that they share precisely that property? Are there not various descriptions under which they fall, and corresponding propositions to the effect that precisely these propositions fall under that description? Is there not a truth to the effect that precisely these propositions are, say, propositions? Given any of these, we can proceed with the argument as before. We need not pause to worry about the vagaries of 'about' because we don't need it, here or in other Cantorian arguments. *That* strategy for denying diagonals, at least, appears to be of insufficient power and generality.

As I've said, I think there are also other problems facing a strategy of denying diagonals. As indicated at an earlier point, it doesn't look like there can be a universal policy for such denials. I think the general problem of the 'reappearing diagonal' also remains.

5. That is at any rate where my thinking stands now. I think you might be right that we are approaching the ends of our respective ropes, and this may be as far as we are equipped to take the issue at the moment.

11. PLANTINGA TO GRIM

Right; I think we should wind this discussion up (or down); we've made some progress, but of course haven't finally settled a whole lot.

With respect to your last letter: on one point you are right: even if, as I suggest, we take omniscience to be maximal perfection with respect to knowledge, it doesn't follow that your Cantorian worries don't pose problems for it. For it might be that what they imply is that there isn't any maximal degree of this perfection. That is surely (as you say) a possibility.

But of course I am still unconvinced that we *have* a genuine Cantorian problem for the claim that

(1) For every proposition **p**, God knows **p** if and only if **p** is true.

The problem, as you see it, attaches to the apparent quantification over all propositions in (1); it is at that point that the alleged Cantorian difficulties raise their ugly heads.

But I am still doubtful that there is a real problem here. We agree, I take it, that there isn't any sound Cantorian argument for the general conclusion that there are no universal propositions (propositions about all propositions); any such argument (according to the argument of my last letter) would involve at least one universal proposition and would thus itself fail to exist if it were sound.

You suggest, however, that there are nevertheless still Cantorian difficulties for (1); we can instead turn to *particular* Cantorian arguments; for any particular claim (such as (1)) that seems to be about all propositions, there will be a particular Cantorian argument making trouble for it. As you put it,

> Now the interesting thing about this core argument is that it is written *in the particular* – it deals simply with a single candidate set **T** or being **B** or proposition **p**. I think, moreover, that it can in each case be written *purely* in the particular, without any universal propositions at all.

And you go on to say that "such a form of argument will continue to cause problems for some of the things you claim to believe . . . ."

Now here I'd like to investigate briefly the claim that "this core argument can be written, in each case, *purely* in the particular, without any universal propositions at all." How would this go, for example, in the case at hand, (1) above? The relevant Cantorian argument, presumably, will be for the conclusion that there just *isn't* any such proposition as (1). How will that argument go? Well, perhaps as follows:

(2) If there is such a proposition as (1), then there is a 1-1 function **f** from the propositions (1) is about onto propositions *simpliciter*;

and

(3) If there is such a function, then there is a proposition **p** about exactly those propositions **p** such that **f(p)** is not about **p**.

Furthermore (the argument continues) the inverse image **r** of **q** under **f** is such that **q** is about **r** if and only if **q** isn't about **r**.

This is how the argument would go; and your suggestion, presumably, is that (2) and (3), the central premises of the argument, are not universal propositions.

That's right or at any rate reasonable: (2) as it stands isn't a universal proposition. Its consequent, however, clearly *involves* quantification over all propositions; the consequent of (2) is or is equivalent to

(2c) There is a 1-1 function **f** such that for any proposition **p**, there is a proposition **q** such that **p** will be the value of **f** for **q** taken as argument.

I suppose we would agree, furthermore, that (2) couldn't so much as exist if its consequent didn't; so (2) couldn't exist if (2c) didn't. But if there is a good Cantorian argument against the existence of (1), there will obviously be an equally good Cantorian argument (one paralleling the argument for the nonexistence of (1)) against the existence of (2c). But (2) can exist only if (2c) does. So if there is a good Cantorian argument for the nonexistence of (1), there is an equally good one for the nonexistence of (2). (Obviously the same will go for (3); both its antecedent and its consequent involve quantification over all propositions.) But (2) is an essential part of the Cantorian argument against (1). So if this (particular) Cantorian argument against (1) is sound, one if its premises doesn't exist; hence it isn't sound.

I am therefore not inclined to think that the move to the particular will help: true, the particular Cantorian argument against (1) needn't itself have a universal proposition as a premise, but its premises will *involve* quantification over all propositions, in the sense that if there are no propositions that are about all propositions, then these premises would not exist.

This has a direct bearing on a second interesting claim you make. You suggest that when confronted with one of these specific Cantorian arguments, I will reject the diagonal premise; thus in the above argument I will, you think, reject (3). Right; I do reject (3); it looks to me as if the proposition **q** proposed, the one that is about exactly those propositions **p** such that **f(p)** is not about **p** — it looks as if that proposition would have to be stated in some such way as follows:

for any proposition **p**, if . . . .

but a proposition like *that* doesn't look to be about only so and so's; it *seems* to be about all propositions. Maybe it isn't obvious that it really *is* about all propositions; but it is certainly far from obvious that it *isn't*. It is therefore at any rate far from clear that there is such a proposition — much less clear that there is such a proposition, e.g., as

God knows, for every proposition **p** whether **p** is true.

Now here is where you make that second interesting claim. You suggest that there are *other* Cantorian arguments against (1), arguments that don't involve the claim that there is a proposition about exactly those propositions that are not about **f(p)**. Well, perhaps there are. As you know, I don't have a perfectly *general* strategy for dealing with Cantorian arguments; I must take them one at a time. In accord with that policy, I'd have to look at these other arguments you say there are, and look at *them* one at a time. That said, however, I must add that it seems likely to me that any such argument will contain a premise *involving*, in the above sense, quantification over all propositions. That is, any such argument will contain a premise **p** which is such that there is a universal proposition **q** so related to it that **p** can't exist unless **q** does — in which case, once more, the proposed argument will be sound only if it doesn't exist.

Thus, for example, you suggest that there is a Cantorian argument against (1) that proceeds in terms of a *property* had by exactly those propositions **p** such that **f(p)** is not about **p** (rather than a *proposition* about exactly those propositions). This argument, I suppose, will retain premise (2) as it stands but replace (3) by something like

(3*) For any function **f**, if **f** is 1-1 and from propositions onto propositions *simpliciter*, then there is a property **q** had by exactly those propositions **p** such that **f(p)** is not about **p**.

And then the argument would proceed.

But of course (2) and (3*) both involve quantification over all propositions; they are therefore such that if there is a sound Cantorian argument for the nonexistence of (1), there will be an equally sound Cantorian argument for the nonexistence of *them*.

By way of brief recapitulation: I argued last time that there aren't any sound Cantorian arguments for the conclusion that there are no universal propositions. You agree, but point out that there may nonetheless be particular Cantorian arguments against the existence of particular universal propositions — (1), for example; and these need not necessarily invoke as premises any universal propositions. Here I think you are right. But (as we have seen) it looks as if such arguments will nevertheless invoke premises which couldn't exist unless universal propositions existed. I also proposed, with respect to the general Cantorian argument for there being no universal propositions, that the diagonal premise (whose consequent affirms the existence of a proposition about just those propositions **p** such that **f(p)** is not about **p**) is surely not obviously true and is quite properly rejectable. In response to this point, you suggest next that there may be *other* Cantorian arguments against the existence of (1) that do not involve the claim that there is a proposition about just those propositions, but instead (for example) endorse the existence of a property had by all and only those propositions). Here my strategy would be, when presented with one of these arguments, to look for a premise like (2) or (3*) — one that isn't itself universal, but is nonetheless such that it couldn't exist if no universal propositions existed. And then the comment on *that* argument would be that if it is sound, then there will be a sound argument against the existence of one of its premises: so it isn't sound.

I therefore remain unconvinced that we have a real problem here for (1); I suspect you remain convinced that we do. No doubt there remains much more to be said on both sides; but perhaps for now you and I have said about all we can usefully say. So we haven't come to agreement; but I have learned much from our discussion, and am grateful to you for having raised the issue.

NOTES

[1] Such an argument also appears in "There is no Set of All Truths," *Analysis* 44 (1984) 206–208 and *The Incomplete Universe*, MIT Press/Bradford Books, 1991.

[2] For the most part what follows is edited from an extended correspondence between the authors. The final two sections, however, are new and were written with this piece in mind.

[3] I'm obliged to Gary Mar for consultation on symbolism.

[4] Patrick Grim, "On Sets and Worlds: A Reply to Menzel," *Analysis* 46 (1986), 186–191.

[5] Keith Simmons actually did argue for something like this position in "On an

Argument against Omniscience," APA Central Division meetings, New Orleans, April 1990.

[6] See G. E. M. Anscombe, "Analysis Puzzle 10," *Analysis* 17 (1957), 49–52, and Susan Haack, "Mentioning Expressions," *Logique et Analyse* 17 (1974), 277–294 and *Philosophy of Logics*, Cambridge Univ. Press, 1978.

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